Einstein, A.
Relativity.

Manuscript.

1948.

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**RELATIVITY. The Essence of the Theory of Relativity.** Mathematics deals exclusively with the relations of concepts of experience. Physics, too, deals with mathematical concepts; however, these concepts attain physical content only by the clear determination of their relation to the objects of experience. This in particular is the case for the concepts of motion, space, time.

~~**Principle of Relativity. There Is No „Absolute Motion.„**~~ The theory of relativity is that physical theory which is based on a consistent physical interpretation of these three concepts. The name "theory of relativity" is connected with the fact that motion, from the point of view of possible experience, always appears as the *relative* motion of one object with respect to another (e.g., of a car with respect to the ground, or the earth with respect to the sun and the fixed stars). Motion is never observable as "motion with respect to space" or, as it has been expressed, as "absolute motion." The "principle of relativity" in its widest sense is contained in the statement: The totality of physical phenomena is of such a character that it gives no basis for the introduction of the concept of "absolute motion"; or, shorter but less precise: There is no absolute motion.

It might seem that our insight would gain little from such a negative statement. In reality, however, it is a strong restriction for the (conceivable) laws of nature. In this sense there exists an analogy between the theory of relativity and thermodynamics. The latter, too, is based on a negative statement: "There exists no perpetuum mobile."

The development of the theory of relativity proceeded in two steps, "special theory of relativity" and "general theory of relativity." The latter presumes the validity of the former and is its consistent continuation.

## Special Theory of Relativity

**Physical Interpretation of Space and Time in Classical Mechanics.** Geometry, from a physical standpoint, is the totality of laws according to which rigid bodies mutually at rest can be placed with respect to each other (e.g., a triangle consists of three rods whose ends touch permanently). It is assumed that with such an interpretation the Euclidean laws are valid. "Space" in this interpretation is in principle an infinite rigid body (or skeleton) to which the position of all other bodies is related (body of reference). Analytic geometry (Descartes) uses as the body of reference, which represents space, three mutually perpendicular rigid rods on which the "co-ordinates" (x, y, z) of space points are measured in the known manner as perpendicular projections (with the aid of a rigid unit-measure).

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# AMERICAN PEOPLES ENCYCLOPEDIA

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$$(1)\quad ds^2 = dx_o^2 + dy_o^2 + dz_o^2 - c^2 dt_o^2$$

is a measurable quantity which is independent of the special choice of the inertial system. If one introduces in this space the new co-ordinates $x_1$, $x_2$, $x_3$, $x_4$ through a general transformation of co-ordinates, then the quantity $ds^2$ for the same pair of points has an expression of the form

$$(2)\quad ds^2 = \sum g_{ik} dx^i dx^k$$

(summed for i and k from 1 to 4) where $g_{ik} = g_{ki}$. The $g_{ik}$ which form a "symmetric tensor" and are continuous functions of $x_1, \ldots, x_4$ then describe according to the "principle of equivalence" a gravitational field of a special kind (namely one which can be retransformed to the form (1)). From Riemann's investigations on metric spaces the mathematical properties of this $g_{ik}$ field can be given exactly ("Riemann-condition"). However, what we are looking for are the equations satisfied by "general" gravitational fields. It is natural to assume that they, too, can be described as tensor-fields of the type $g_{ik}$.

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**AMERICAN PEOPLES ENCYCLOPEDIA**

Vol. Page Col.

Article RELATIVITY Original Writer Editor Copy Sheet Date

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The general theory of relativity is as yet incomplete insofar as while it leads to a well-defined theory of the gravitational field it does determine sufficiently the theory of the total field (which includes the electromagnetic field). The reason for this is the fact that the general field laws are not sufficiently determined by the general principle of relativity alone.

Albert Einstein

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**History.** Joseph Monier, a Parisian gardener, is often credited with the introduction of reinforced concrete, but the idea of using steel and iron to supplement the strength of concrete was not the product of a single mind. Scores of engineers and craftsmen took part in the early experiments and later developments.

The first experiment of which there is a complete record was conducted by Sir MARC ISAMBARD BRUNEL in London, England, in 1832. Brunel was building a tunnel under the Thames River and built an experimental arch of brick and cement, using strips of hoop-iron and wood for reinforcement. All early development work was done in Europe and Great Britain. The use of reinforced concrete for walls, roofs, beams, and arches had progressed far before much attention was paid to its use in America. Monier, perhaps the best known of the early experimenters, received a patent in 1867 on a method of reinforcing concrete with iron wire mesh. Other inventors patented various methods.

In the United States, among the earliest examples of experimental construction with reinforced concrete is a house in Port Chester, N.Y., built by W. E. Ward in 1871 and 1872. Interior and exterior walls, floors, and roof were built of concrete, the bottom of the beams being reinforced with light I beams. The first reinforced concrete bridge in the United States is believed to have been a small one built in Prospect Park in Brooklyn in 1871.

**Developments and Applications.** The first important practical applications of reinforced concrete for building construction were promoted on the Pacific coast by E. L. Ransome, beginning about 1874. He first used old wire cable and hoop-iron for reinforcement. In 1884 he took out a patent on a deformed bar, made by twisting a square bar. He built the famous Leland Stanford, Jr. Museum at Palo Alto, which came through the 1906 earthquake with minor damage. An addition to the borax works at Alameda, Calif., which Ransome built in 1889, is said to be the first example of ribbed floor construction in America.

Joseph Melan, an Austrian, was the inventor of a system of reinforcing concrete with specially designed structural shapes; his system was introduced into the United States by an Austrian associate, Fritz Emperger, about 1893. A paper describing the Melan system, delivered by Emperger before the American Society of Civil Engineers in 1894, is said to be the first detailed technical description of the use of reinforced concrete for bridges ever given in this country.

One of the most notable of all reinforced concrete bridges is the Tunkhannock Viaduct, among the largest concrete bridges in the world. It is 2,375 feet long, rises 300 feet above its foundations, bedded in solid rock, and required 165,000 cubic yards of concrete. This bridge was built in 1912–1914.

Other notable reinforced concrete bridges in the United States include the Capplen Memorial Arch Bridge at Minneapolis; the Bay Bridge at San Francisco; the Lake Pontchartrain Bridge in Louisiana; the James River Bridge at Newport News, the Two Mile Viaduct at Long Key, Fla., and the Lake Washington Pontoon Bridge at Seattle, Wash. The five-span George Westinghouse Bridge at East Pittsburgh, Pa., has one of the longest concrete arches in the country, 460 feet with a rise of 156 feet. But increase in the number and size of American bridges is not the most important contribution to bridgebuilding resulting from the development of reinforced concrete. Equally important has been improvement in the architectural design of bridges, made possible by the use of concrete that can be made to assume any intricate shape, form, or texture conceived by the designer. Improvement in the architectural appearance of bridges has been most notable during the past 25 years.

The first skyscraper type reinforced concrete building to be erected in the United States was the 16-story Ingalls Building in Cincinnati, Ohio, designed by William P. Anderson and completed in 1903. Original plans for the building had called for structural steel construction, but slow deliveries of steel led to acceptance of Anderson's proposal to use reinforced concrete. Since that time thousands of large reinforced concrete buildings have been constructed in every part of the country, including some of the largest schools, hospitals, hotels, public buildings, office buildings, and factories in the world. A number of the great dams such as Hoover Dam and Grand Coulee Dam are of reinforced concrete.

HERBERT C. PERSONS

BIBLIOG.–H. Sutherland and R. C. Reese, *Introduction to Reinforced Concrete Design* (1943); G. A. Hool and W. S. Kinne, *Reinforced Concrete and Masonry Structures* (1944); W. S. Gray, *Reinforced Concrete Water Towers, Bunkers, Silos, and Gantries* (1944); C. W. Dunham, *Theory and Practice of Reinforced Concrete* (1944).

**REINHARDT, MAX,** real name **Max Goldmann,** 1873–1943, Austrian theatrical producer, was born in Baden. Beginning as a character actor, he became manager of the Kleine Theater's cabaret *Schall und Rauch* and director of the Neues Theater by 1902. In 1905, as director of the Kammerspiele and the Deutsches Theater, he began to make stage history with his impressionistic productions of Shakespeare, Molière, Gorki, Wilde, Strindberg, Wedekind, and Shaw. In the summer he managed the Salzburg Festspielhaus, producing each season the old mystery play, *Everyman.* His production of Karl Vollmöeller's pantomime, *The Miracle*, in 1912 established his reputation internationally; and when the Nazis forced him to leave Germany in 1932, he went to London to stage *A Midsummer Night's Dream* for the Oxford University Dramatic Society. Two years later he went to America to supervise the film version of *A Midsummer Night's Dream* and in 1937 he produced Franz Werfel's *The Eternal Road* in New York.

**REJECTION, PARENTAL.** See PARENTHOOD.

**RELATIVE HUMIDITY, PERCENTAGE OF,** a term indicating the moisture content of any atmosphere per unit weight as compared to the total amount that could be contained by the atmosphere at its particular temperature. Saturation at any temperature is given as 100 per cent. In contrast, *absolute humidity* indicates the moisture content in grains of moisture per cubic foot of dry air. See AIR CONDITIONING; HEATING AND VENTILATION; HUMIDITY; PSYCHROMETRY.

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**Simultaneity.** Physics deals with "events" in space and time. To each event belongs, besides its place co-ordinates x, y, z, a time value t. The latter was considered measurable by a clock (ideal periodic process) of negligible spatial extent. This clock C is to be considered at rest at one point of the co-ordinate system, e.g., at the co-ordinate origin $(x=y=z=0)$. The time of an event taking place at a point P(x, y, z) is then defined as the time shown on the clock C simultaneously with the event. Here the concept "simultaneous" was assumed as physically meaningful without special definition. This is a lack of exactness which seems harmless only since, with the help of light (whose velocity is practically infinite from the point of view of daily experience), the simultaneity of spatially distant events can apparently be decided immediately.

The special theory of relativity removes this lack of precision by defining simultaneity physically with the use of light signals. The time t of the event in P is the reading of the clock C at the time of arrival of a light signal emitted from the event, corrected with respect to the time needed for the light signal to travel the distance. This correction presumes (postulates) that the velocity of light is constant.

This definition reduces the concept of simultaneity of spatially distant events to that of the simultaneity of events happening at the same place (coincidence), namely the arrival of the light signal at C and the reading of C.

**Inertial Systems and the L-Principle.** Classical mechanics is based on Galileo's principle: A body is in rectilinear and uniform motion as long as other bodies do not act on it. This statement cannot be valid for arbitrary moving systems of co-ordinates. It can claim validity only for so-called "inertial systems." Inertial systems are in rectilinear and uniform motion with respect to each other. In classical physics laws claim validity only with respect to all inertial systems (special principle of relativity).

It is now easy to understand the dilemma which has led to the special theory of relativity. Experience and theory have gradually led to the conviction that light in rmpty space always travels with the same velocity c independent of its color and the state of motion of the source of light (principle of the constancy of the velocity of light—in the following referred to as "L-principle"). Now elementary intuitive considerations seem to show that the same light ray *cannot* move with respect to all inertial systems with the same velocity c. The L-principle seems to contradict the special principle of relativity.

It turns out, however, that this contradiction is only an apparent one which is based essentially on the prejudice about the absolute character of time or, rather, of the simultaneity of distant events. We just saw that x, y, z and t of an event can, for the moment, be defined only with respect to a certain chosen system of co-ordinates (inertial system). The transformation of the x, y, z, t of events which has to be carried out with the passage from one inertial system to another (co-ordinate transformation), is a problem which cannot be solved without special physical assumptions. However, the following postulate is exactly sufficient for a solution: *The L-principle holds for all inertial systems* (application of the special principle of relativity to the L-principle). The transformations thus defined, which are linear in x, y, z, t, are called Lorentz transformations. Lorentz transformations are formally characterized by the demand that the expression

$$dx^2+dy^2+dz^2-c^2dt^2,$$

which is formed from the co-ordinate-differences dx, dy, dz, dt of two infinitely close events, be invariant (i.e., that through the transformation it goes over into the *same* expression formed from the co-ordinate differences in the new system).

With the help of the Lorentz transformations the special principle of relativity can be expressed thus: The laws of nature are invariant with respect to Lorentz-transformations (i.e., a law of nature does not change its form if one introduces into it a new inertial system with the help of a Lorentz-transformation on x, y, z, t).

**Results of the Special Theory of Relativity.** The special theory of relativity has led to a clear understanding of the physical concepts of space and time and in connection with this to a recognition of the behavior of moving measuring rods and clocks. It has in principle removed the concept of absolute simultaneity and thereby also that of ~~distant force~~ in the sense of Newton. It has shown how the law of motion must be modified in dealing with motions that are not negligibly small as compared with the velocity of light. It has led to a formal clarification of Maxwell's equations of the electromagnetic field; in particular it has led to an understanding of the essential oneness of the electric and the magnetic field. It has unified the laws of conservation of ~~impulse~~ and of energy into one single law and has demonstrated the equivalence of mass and energy. From a formal point of view one may characterize the achievement of the special theory of relativity thus: it has shown generally the role which the universal constant c (velocity of light) plays in the laws of nature and has demonstrated that there exists a close connection between the form in which time on the one hand and the spatial co-ordinates on the other hand enter into the laws of nature.

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## General Theory of Relativity

The special theory of relativity retained the basis of classical mechanics in one fundamental point, namely the statement: The laws of nature are vlid only with respect to inertial systems. The "permissible" transformations for the co-ordiqates (i.e., those which leave the form of the laws unchanged) are *exclusively* the (linear) Lorentz-transformations. Is this restriction really founded in physical facts? The following argument convincingly denies it.

**Principle of Equivalence.** A body has an inertial mass (resistance to acceleration) and a heavy mass (which determines the weight of the body in a given gravitational field, e.g., that at the surface of the earth). These two quantities, so different according to their definition, are according to experience measured by one and the same number. There must be a deeper reason for this. The fact can also be described thus: In a gravitational field different masses receive the same acceleration. Finally, it can also be expressed thus: Bodies in a gravitational field behave as in the absence of a gravitational field if, in the latter case, the system of reference used is a uniformly accelerated co-ordinate system (instead of an inertial system).

There seems, therefore, to be no reason to ban the following interpretation of the latter case. One considers the system as being "at rest" and considers the "apparent" gravitational field which exists with respect to it as a "real" one. This gravitational field "generated" by the acceleration of the co-ordinate system would of course be of unlimited extent in such a way that it could not be caused by gravitational masses in a finite region; however, if we are looking for a fieldlike theory, this fact need not deter us. With this interpretation the inertial system loses its meaning and one has an "explanation" for the equality of heavy and inertial mass (the same property of matter appears as weight or as inertia depending on the mode of description).

**Enlargement of the Principle of Relativity.** Considered formally, the admission of a co-ordinate system which is accelerated with respect to the inertial origin of co-ordinates means the admission of nonlinear co-ordinate transformations, hence a mighty enlargement of the idea of invariance ,i.e., the principle of relativity.

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First, a penetrating discussion, using the results of the special theory of relativity, shows that with such a generalization the co-ordinates can no longer be interpreted directly as the results of measurings. Only the co-ordinate difference together with the field quantities which describe the gravitational field determine measurable distances between events. After one has found oneself forced to admit nonlinear co-ordinate transformations as transformations between equivalent co-ordinate systems, the simplest demand appears to admit all continuous co-ordinate transformations (which form a group), i.e., to admit arbitrary curvilinear co-ordinate systems in which the fields are described by regular functions (general principle of relativity).

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**Gravitation in the General Theory of Relativity.** Now it is not difficult to understand why the general principle of relativity (*on the basis of the equivalence principle*) has led to a theory of gravitation. There is a special kind of space whose physical structure (field) we can presume as precisely known on the basis of the special theory of relativity. This is empty space without electromagnetic field and without matter. It is completely determined by its "metric" property: Let dx., dy., dz1, dt1 be the co-ordinate differences of two infinitesimally near points (events); then

(1) ds/ = dx:/ + dy:/ + dz:/ − c/dt:/

is a measurable quantity which is independent of the special choice of the inertial system. If one introduces in this space the new co-ordinates x/, x/, x/, x/ through a general transformation of co-ordinates, then the quantity ds/ for the same pair of points has an expression of the form

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**Experimental VeriLcations of the General Theory of Relativity.** These equations yield Newton's equations of gravitational mechanics as an approximate law and in addition certain small effects which have been confirmed by observation (deflection of light by the gravitational field of a star, influence of the gravitational potential on the frequency of emitted light, slow rotation of the elliptic circuits of planets—perihelian motion of the planet Mercury). They further yield an explanation for the expanding motion of galactic systems, which is manifested by the red-shift of the light emitted from these systems.

~~The general theory of relativity is as yet incomplete~~ insofar as it has been able to apply the general principle of relativity satisfactorily only to gravitational fields, but not to the total field. We do not yet know, with certainty, by what mathematical mechanism the total field in space is to be described and what the general invariant laws are to which this total field is subject. One thing, however, seems certain: namely, that the general principle of relativity will prove a necessary and effective tool for the solution of the problem of the total field.

ALBERT EINSTEIN

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tr $$(1)\ ds^2 = dx_0^2 + dy_0^2 + dz_0^2 - c^2dt_0^2$$ tr

is a measurable quantity which is independent of the special choice of the inertial system. If one introduces in this space the new coördinates $x_1, x_2, x_3, x_4$ through a general transformation of coördinates, then the quantity $ds$ for the same pair of points has an expression of the form

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(summed for i and k from 1 to 4) where $g_{ik} = g_{ki}$. The $g_{ik}$ which form a "symmetric tensor" and are continuous functions of $X_1, \ldots, x_4$ then describe according to the "principle of equivalence" a gravitational field of a special kind (namely one which can be transformed to the form (1)). From Riemann's investigations on metric spaces the mathematical properties of this $g_{ik}$ field can be given exactly ("Riemann-condition"). However, what we are looking for are the equations satisfied by "general" gravitational fields. It is natural to assume that they, too, can be described as tensor-fields of the type $g_{ik}$ which in general do *not* admit a transformation to the form (1), i.e., which do not satisfy the "Riemann condition," but weaker conditions, which, just as the Riemann condition, are independent of the choice of co-ordinates (i.e., are generally invariant). A simple formal consideration leads to weaker conditions which are closely connected with the Riemann condition. These conditions are the very equations of the pure gravitational field (on the outside of matter and at the absence of an electromagnetic field).

**Experimental Verifications of the General Theory of Relativity.** These equations yield Newton's equations of gravitational mechanics as an approximate law and in addition certain small effeets which have been confirmed by observation (deflection of light by the gravitational field of a star, influence of the gravitational potential on the frequency of emitted light, slow rotation of the elliptic circuits of planets—perihelian motion of the planet Mercury). They further yield an explanation for the expanding motion of galactic systems, which is manifested by the red-shift of the light emitted from these systems.

The general theory of relativity is as yet incomplete insofar as while it leads to a well-defined theory of the gravitational field it does determine sufficiently the theory of the total field (which includes the electromagnetic field). The reason for this is the fact that the general field laws are not sufficiently determined by the general principle of relativity *alone*.

ALBERT EINSTEIN

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**Gravitation in the General Theory of Relativity.** Now it is not difficult to understand why the general principle of relativity (*on the basis of the equivalence principle*) has led to a theory of gravitation. There is a special kind of space whose physical structure (field) we can presume as precisely known on the basis of the special theory of relativity. This is empty space without electromagnetic field and without matter. It is completely determined by its "metric" property: Let $dx_0$, $dy_0$, $dz_0$, $dt_0$ be the co-ordinate differences of two infinitesimally near points (events); then

$$(1)\quad ds^2 = dx_0^2 + dy_0^2 + dz_0^2 - c^2dt_0^2$$

is a measurable quantity which is independent of the special choice of the inertial system. If one introduces in this space the new co-ordinates xl, xl, xl, xl through a general transformation of co-ordinates, then the quantity $ds^2$ for the same pair of points has an expression of the form

$$(2)\quad ds^2 = \Sigma g_{ik}dx^i dx^k$$

(summed for i and k from 1 to 4) where $g_{ik} = g_{ki}$. The $g_{ik}$ which form a "symmetric tensor" and are continuous functions of xl, . . . , xl then describe according to the "principle of equivalence" a gravitational field of a special kind (namely one which can be retransformed to the form (1)). From Riemann's investigations on metric spaces the mathematical properties of this $g_{ik}$ field can be given exactly ("Riemann-condition"). However, what we are looking for are the equations satisfied by "general" gravitational fields. It is natural to assume that they, too, can be described as tensor-fields of the type $g_{ik}$, which in general do *not* admit a transformation to the form (1), i.e., which do not satisfy the "Riemann condition," but weaker conditions, which, just as the Riemann condition, are independent of the choice of co-ordinates (i.e., are generally invariant). A simple formal consideration leads to weaker conditions which are closely connected with the Riemann condition. These conditions are the very equations of the pure gravitational field (on the outside of matter and at the absence of an electromagnetic field).

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Albert Einstein